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## **DIPOLE – DIPOLE ARRAY CONFIGURATION AND INVERSION IN THE FRAMEWORK OF THE RECIPROCITY PRINCIPLE**

A.Fraseri<sup>1</sup>, P. Alikaj<sup>1</sup>, N.Fraseri<sup>2</sup>

<sup>1</sup> Faculty of Geolgy and Mining, Polytechnic University of Tirana, Albania

<sup>2</sup> Faculty of Technology of Informatics, Polytechnic University of Tirana, Albania

### **Abstract**

The dipole-dipole array configuration is considered as a symmetrical array in terms of the reciprocity principle. Aspects of IP data inversion theory are considered, as well as resolution capability and stability of inversion solutions are discussed. This analyze, demonstrates cases when the IP/Resistivity anomaly configurations observed with a  $C_1C_2$ - $P_1P_2$  (AB-MN) array is not the same as the one observed with a  $P_1P_2$ - $C_1C_2$  (MN-AB) reversed array. The analysis includes results of some 2D and 3D mathematical and physical modeling performed in the Institute of Informatics and Applied Mathematics, and in the "Ligor Lubonja" Laboratory of Geophysics at the Faculty of Geology and Mining, Polytechnic University of Tirana, Albania.

### **1. INTRODUCTION**

In the practice of electrical prospecting are employed various array configurations. The location of the current and potential electrodes is defined from the geological tasks to be solved. The dipole – dipole array is one of the most common arrays in mineral exploration. This is considered a symmetrical array in terms of the principle of reciprocity, so when the current electrodes are respectively switched with potential electrodes the same responses in IP and resistivity values are observed. However, our recent mathematical and scale models indicate discrepancies in this regard in several cases. This can lead to inaccurate target location and negative drilling results. To avoid such situations, the electrode orientation in the survey line has to be considered in the interpretation.

### **2. PRESENTATION OF THE PROBLEM**

The well-known reciprocity principle stands on the basis of many array configurations in electrical prospecting like Pole - Pole, Dipole - Dipole, Schlumberger, Wenner etc (Keller and Frischknecht 1966, Zabarovsky 1943, 1963, Fraseri et al. 1985). “According to the theorem of the reciprocity, no changes will be observed in the measured voltage if the

placements of potential and current electrodes are interchanged. The reciprocity can readily be confirmed for an electrode array over a homogeneous earth” (Keller and Frischknecht 1966).

The heterogeneous medium presents a more complicated problem. Zabarovskyy (1943, 1963) based on the electrostatic phenomena science has been observed:

$$U_M = U_A = \alpha_{AM} \cdot Q_A = \alpha_{MA} \cdot Q_M$$

Where:  $Q_A, Q_M$  - Electrical charges

$\alpha_{AM}$  ,  $\alpha_{MA}$  - Coefficients dependant on the shape of bodies A and M, their reciprocal position and the boundaries of heterogeneity.

and equation  $Q_M=Q_A$  will be true if coefficients  $\alpha_{AM} = \alpha_{MA}$ . On this basis Zabarovskyy (1943, 1963) has accepted that the principle of reciprocity is valid for heterogeneous media as well. Habberjam, G.M. (1967), doubt has been expressed about the validity of the reciprocity principle, from field experiments. Reciprocity principle has been discussed by Parasnis D.S. (1988), which has been observed: “Although the reciprocity theorem is often mentioned in books and papers on d.c. resistivity prospecting as well as in books on applied geophysics, no proof of it arbitrary conductivity distribution has, to the best of my knowledge, been given in geophysical literature”. For vertical targets of thickness  $d > a$  ( $a$  stands for dipole spacing) the principle of reciprocity is met while for  $d$  comparable and thinner than  $a$ , the asymmetry is noticed in intensity and shape of the twin responses (Keller and Frischknecht 1966, Frasher et al 1985).

In homogeneous or linear media, as example 2D horizontally stratified section the principle of reciprocity is true for any surveying array. In a heterogeneous environment this principle is absolutely true for symmetrical four electrodes Schlumberger, Wenner and pole-pole (half-Wenner) arrays.

The dipole-dipole array presents a complex behavior. In IP method the principle of reciprocity application is more complicated. In several field surveys asymmetrical IP/Resistivity responses are observed with dipole – dipole array for opposite orientations of the potential and current electrodes in the survey line. To further investigate this phenomenon some mathematical models were carried out with a program of finite element method (Frasher A. and Frasher N. 2000). The mathematical computation of the IP effect is based on the Bleil 1953 and Seigel 1959 formulae. To perform the mathematical modeling and the inversion of IP data, we have used the Komarov’s (1972) approach. For 3D modeling of IP effect from targets with massive texture in homogeneous medium we have transformed the Bleil formulae, using Green’s formulae (Frasher N. 1983, Frasher A., Frasher N. 2000). With the same method of finite elements, simultaneously with the IP effect, the apparent resistivity is calculated as well. Testing of the results of a mathematical IP models with a similar field situation and scale model indicates the accuracy of mathematical model is good (Fig. 1, 2, 3, 4, 5) (Frasher A. 1989, Frasher A. et al. 1994, Frasher A and Frasher N, 2000).

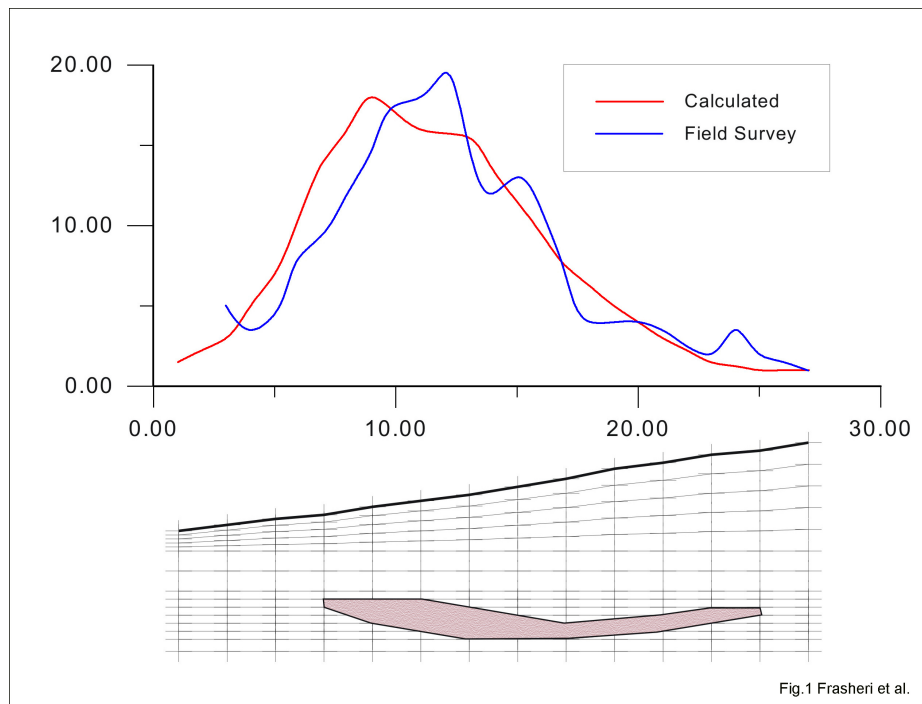


Fig. 1. A finite element section of an IP irregular body over a rugged relief.

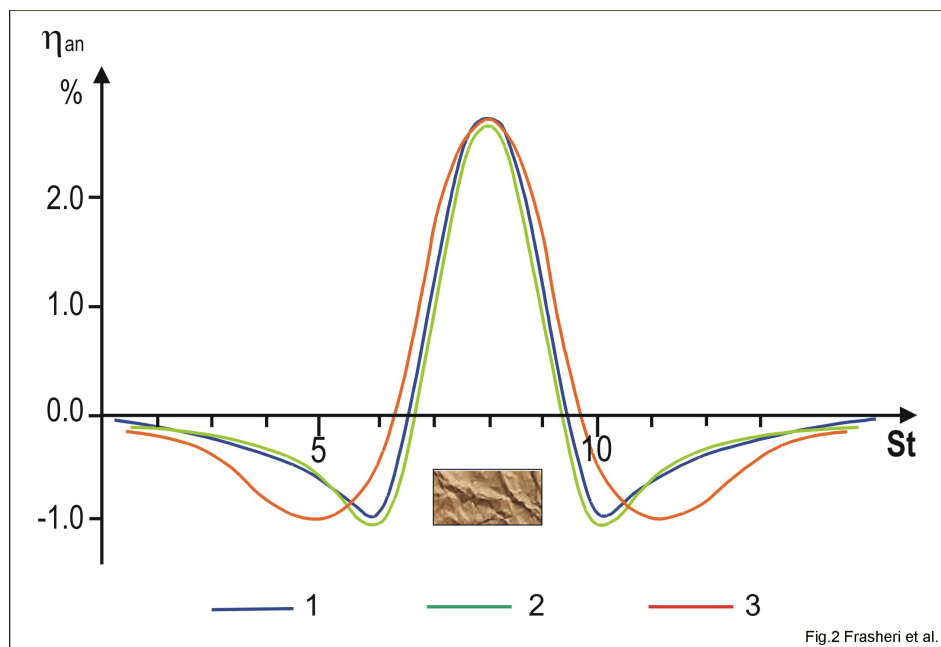


Fig. 2. IP profiling over a prism:  
 Theoretical (1);  
 Calculated by POLARELF Program (2); and  
 Physical modeling (3).

### NORMAL ELECTRIC FIELD PROPAGATION SECTIONS

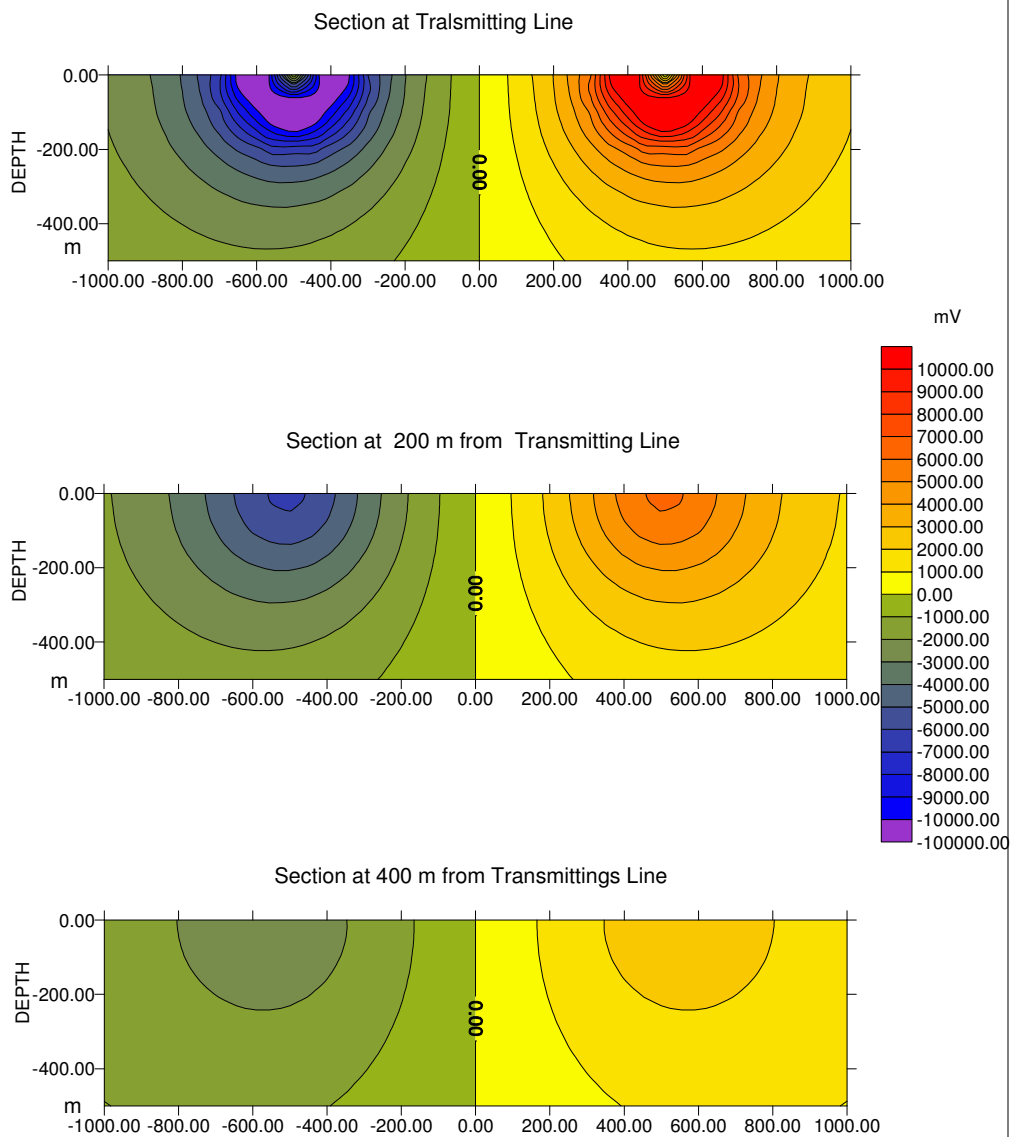
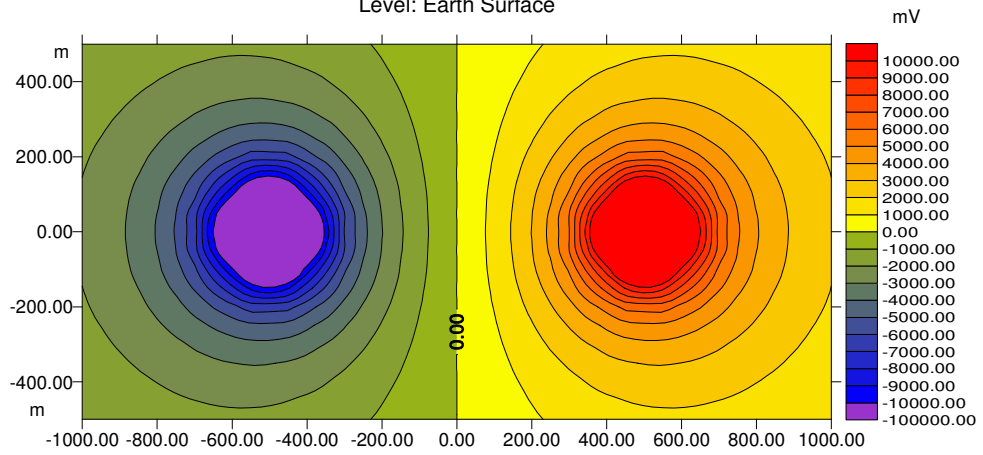


Fig. 3

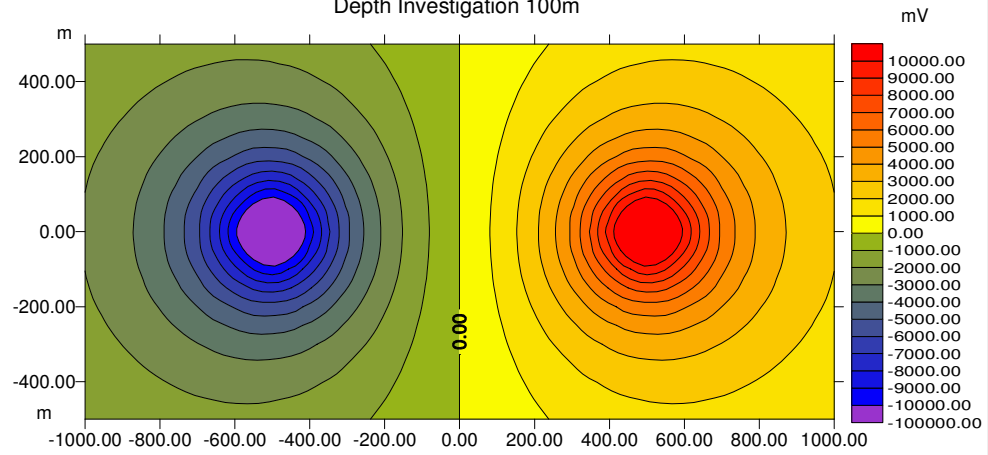
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### MAP OF NORMAL ELECTRIC FIELD PROPAGATION Lower Half-Space Homogeneous Isotropic Medium

Level: Earth Surface



Depth Investigation 100m



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Prof. Dr.; N. Frasher  
September 29, 1998

Fig. 4

MAP OF NORMAL ELECTRIC FIELD PROPAGATION  
Lower Half-Space Homogeneous Isotropic Medium

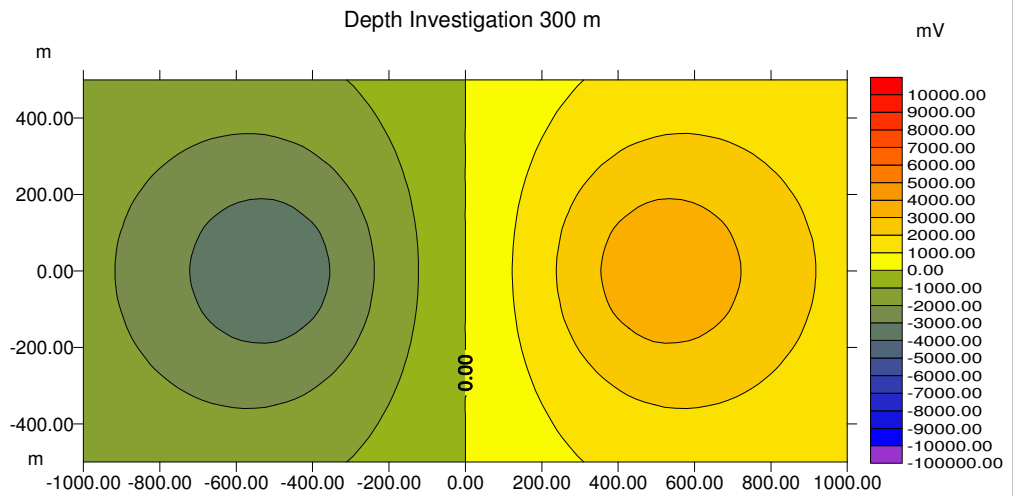
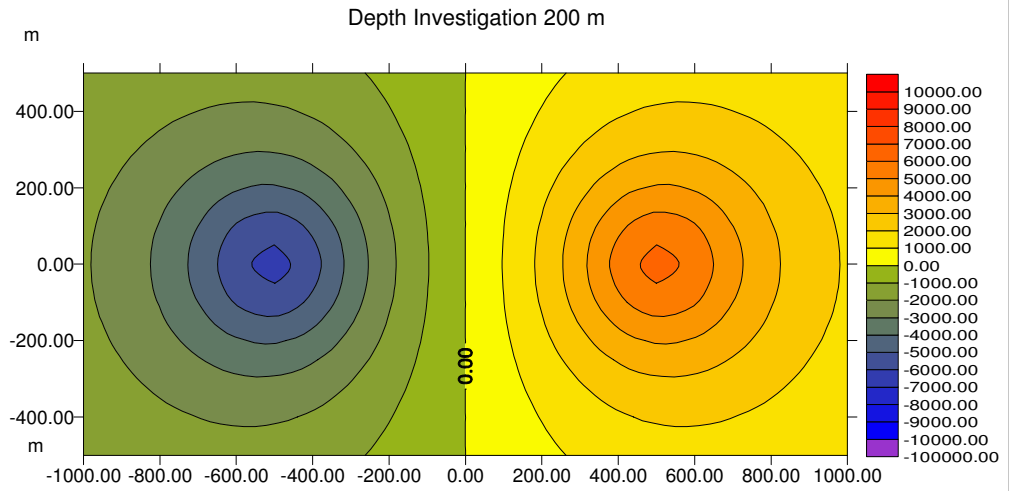


Fig. 5

The amplitude and the asymmetry of IP anomaly depend on the orientation of the polarizing vector of the primary electric field in connection to the prism location (Figs. 6, 7). The substantial difference between the electric field distributions in both cases clearly expresses the changes in IP anomaly configurations for gradient and dipole-dipole arrays.

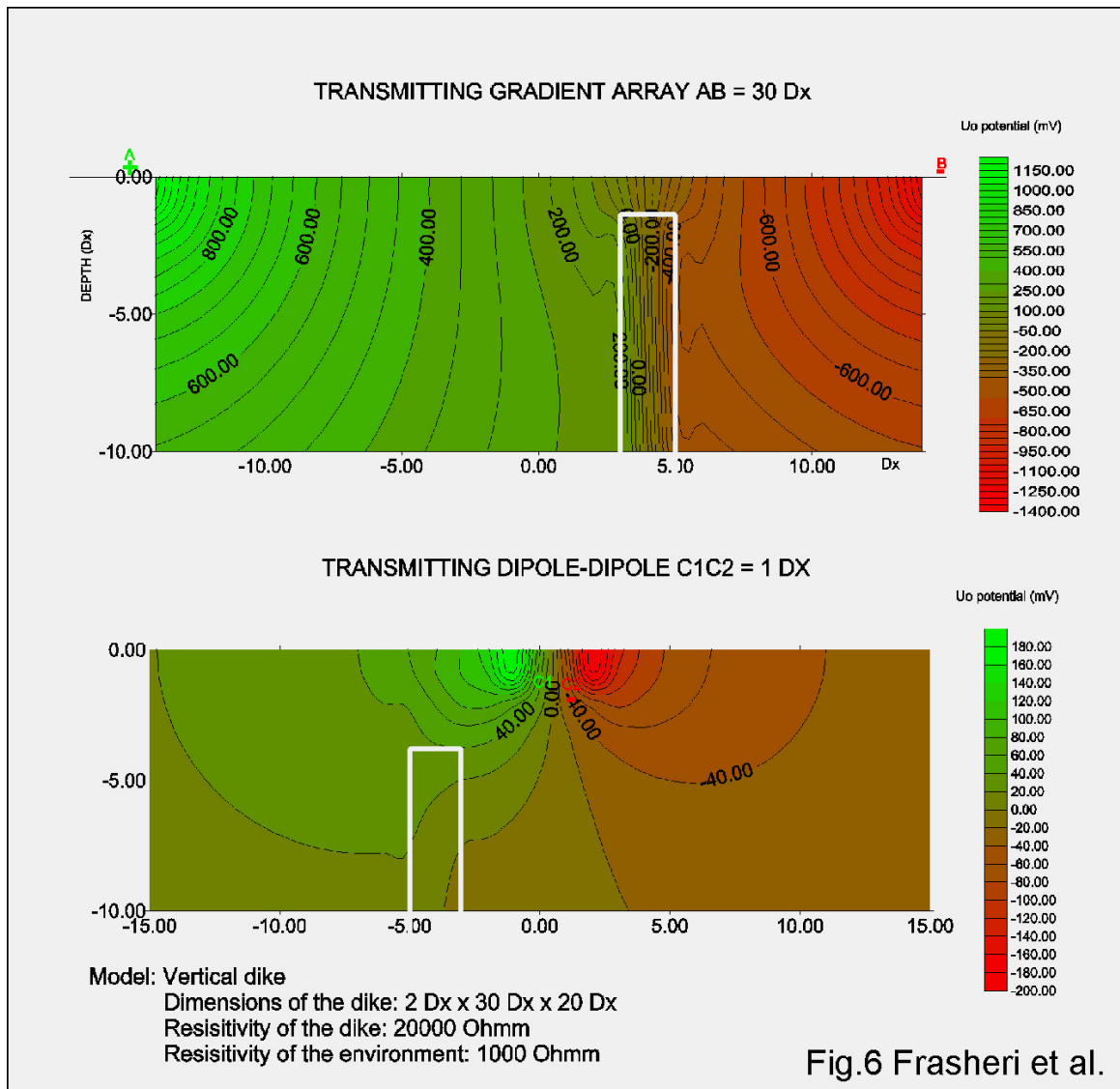


Fig. 6. Distribution of the primary electric field potential ( $U_0$ ) of a transmitting dipole:

- (a) Gradient array  $AB_{\max} = 30 \text{ Dx}$
- (b) Dipole-dipole array  $C_1C_2 = 1 \text{ Dx}$ .

Mathematical model: Vertical prism. Dimensions of the prism  $2 \times 30 \times 20 \text{ Dx}$ , Resistivity of the prism 20,000 Ohmm, Resistivity of the environment 1,000 Ohmm.

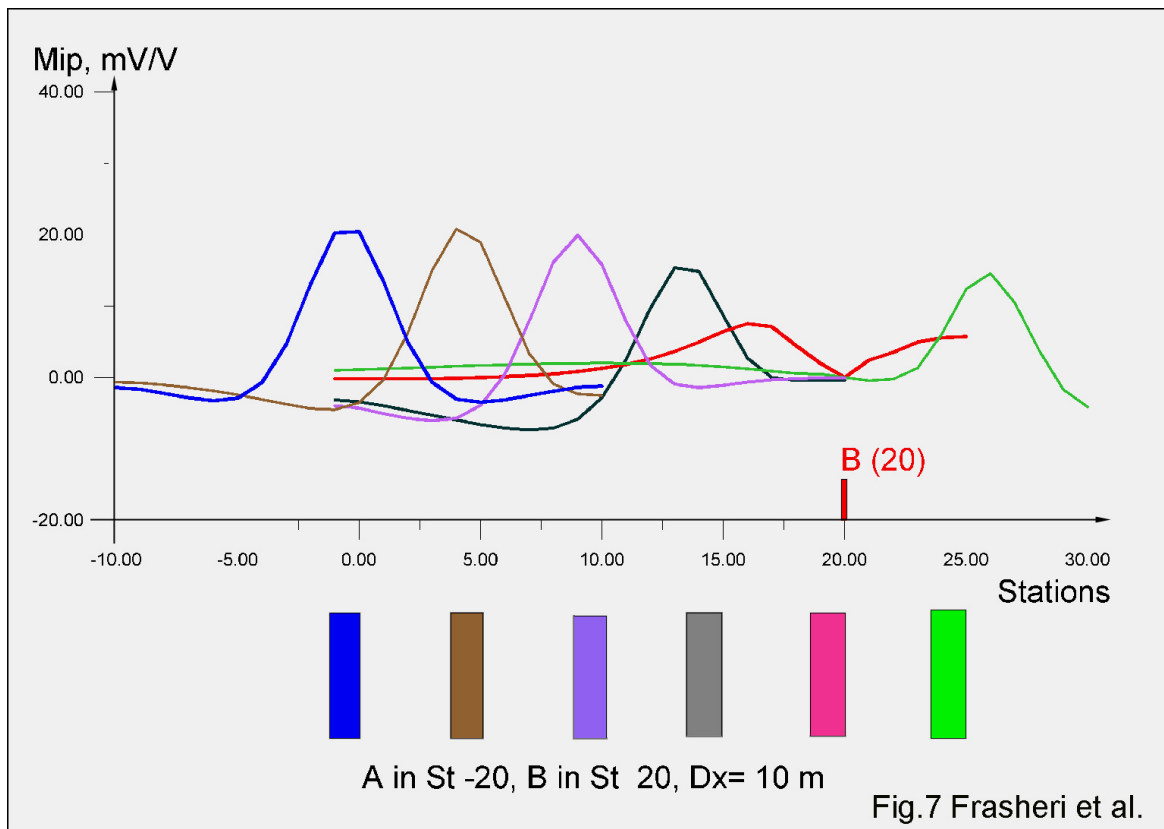


Fig. 7. IP anomaly configuration dependency on location of the target.  
Mathematical model: Vertical prism.

### 3. NUMERICAL RESULTS FOR DIFFERENT MODELS

Fig. 8 present the mathematical model results of IP and resistivity responses with dipole-dipole profiling. Two anomalies are observed on both parameters. Considering the reference plotting point in between the potential electrodes  $P_1$  and  $P_2$ , one of the anomalies is obtained over the prism while the second one at a distance  $O_1O_2$ , between the centers of the current and potential dipoles. This presentation is conditioned on the distribution of the electrical field of the dipole - dipole array.

*Because a mirror image is missing in the center of the profiles, especially for IP, it means that  $C_1C_2P_1P_2$  array responses are not equivalent with  $P_1P_2C_1C_2$ , or in mathematical terms, the principle of reciprocity is not strictly met.* Keller (1966) presents the same phenomenon for the apparent resistivity.



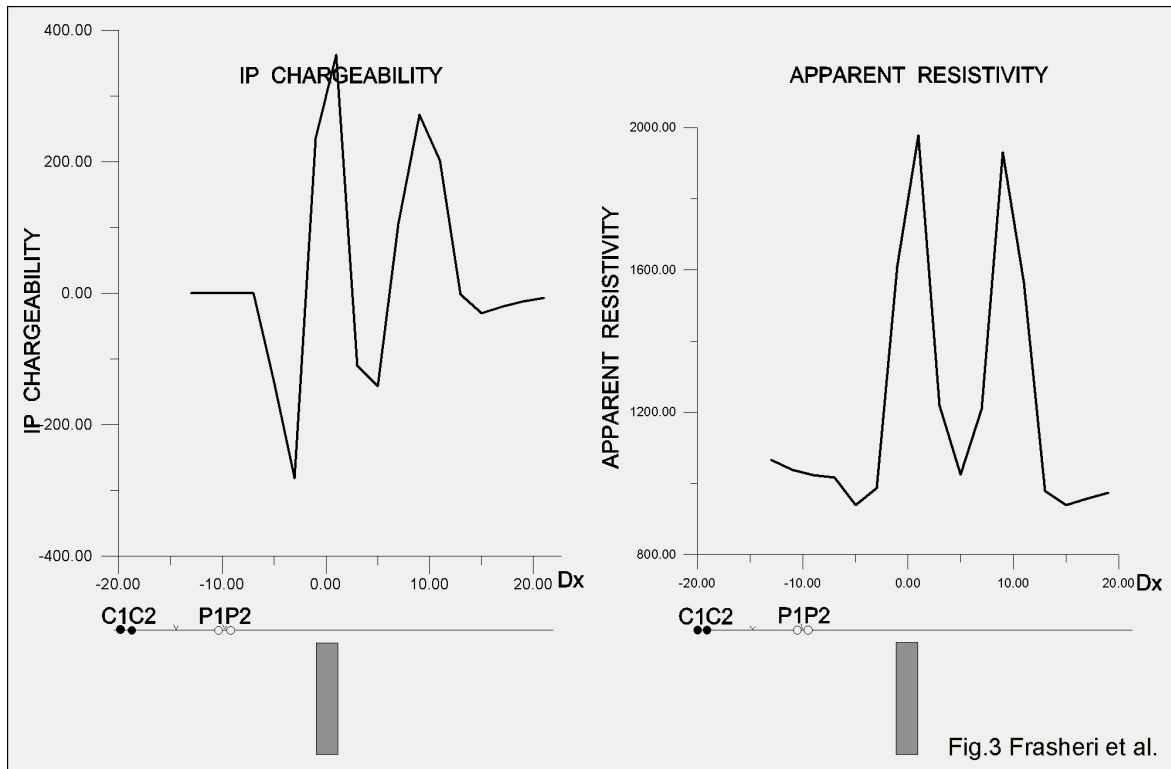


Fig. 8. IP and Resistivity mathematical modeling. Dipole-dipole profiling.  $C_1C_2$ - $P_1P_2=2$  Dx,  $n=1-10$  Dx.

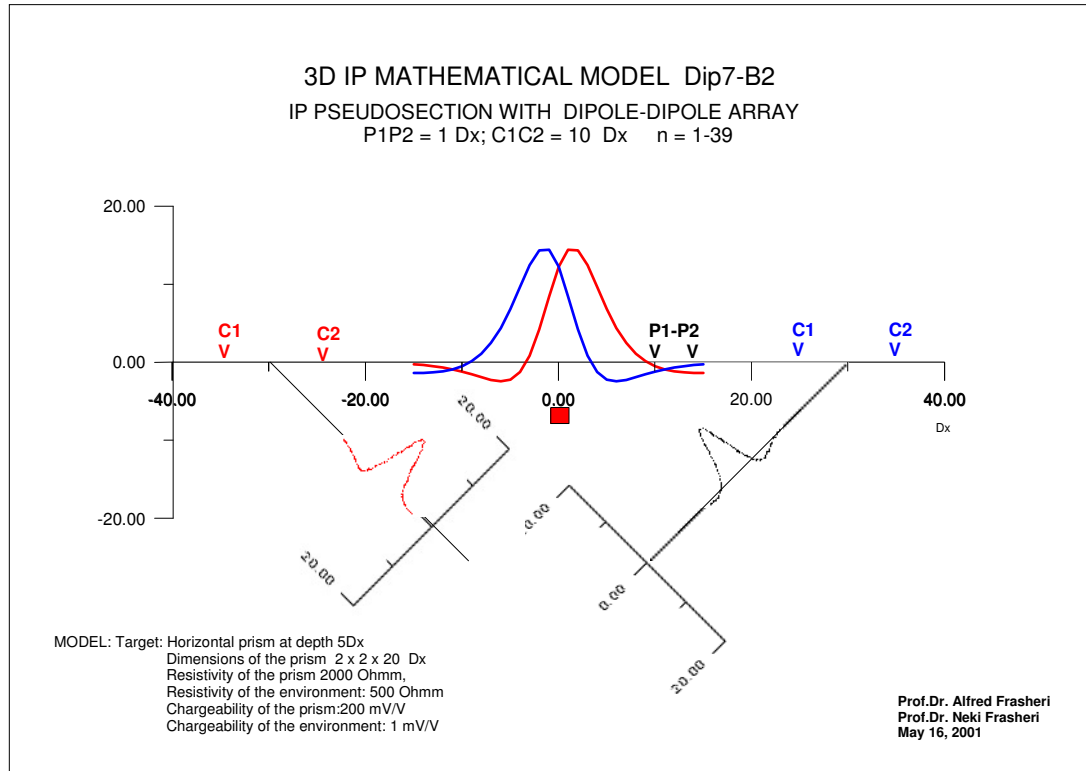
Model: 2D vertical prism at depth 1 Dx, dimensions of the prism section  $2 \times 9$  Dx. Resistivity of the prism 20,000 Ohmm, IP Chargeability 500 mV/V, Resistivity of the environment 1,000 Ohmm, IP Chargeability of the environment 0.01 mV/V.

In pseudo section presentation, where the plotting point is located at the intersection of lines coming at  $45^\circ$  from midpoints between  $C_1C_2$  and  $P_1P_2$ , these anomalies are located in both sides of the prism (Fig. 9, 10). For the resistivity parameter this location is almost symmetrical in shape and amplitude, for the vertical target (Fig:11, 12). The symmetry is perfect in cases when the thickness of the prism is equal or greater than the dipole spacing "a", and becomes poor for thinner prisms (Fig. 11).

Alternatively, the IP anomalies are asymmetrical even in cases of vertical prisms (Fig. 11-a). In such cases, the epicenter of the most intensive anomaly is displaced on the side of current dipole  $C_1C_2$ . For shallow inclined prisms, the epicenters of both IP and resistivity anomalies are displaced on the opposite side of the dip.

The configuration of the IP/Resistivity anomaly is also dependent on the dip angle amplitude, relative to the current electrodes location (Fig. 11-b, c).

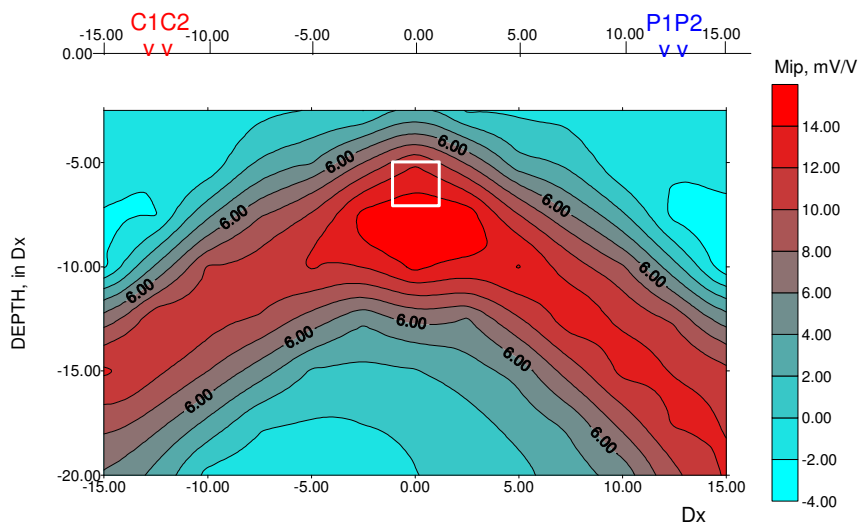
Asymmetrical IP and resistivity anomalies, depending on the location of current and potential dipoles in relation to target is not always without problems in manual or inversion interpretations of the IP/Resistivity data surveyed with a dipole-dipole array.



**Fig. 9**

Fig. 3

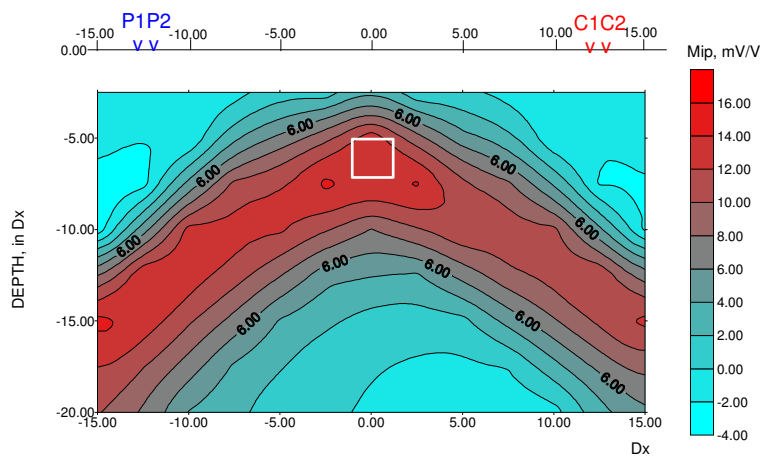
3D IP MATHEMATICAL MODEL Dip7-B2  
 IP PSEUDOSECTION WITH DIPOLE-DIPOLE ARRAY  
 $P1P2 = C1C2 = 1 \text{ Dx}$   $n = 1-39$



MODEL: Target: Horizontal prism at depth 5Dx  
 Dimensions of the prism  $2 \times 2 \times 20 \text{ Dx}$   
 Resistivity of the prism 2000 Ohmm,  
 Resistivity of the environment: 500 Ohmm  
 Chargeability of the prism: 200 mV/V  
 Chargeability of the environment: 1 mV/V

Prof. Dr. A. Frasheri  
 Prof. Dr. N. Frasheri  
 May 28, 2001

3D IP MATHEMATICAL MODEL Dip7-B2  
 IP PSEUDOSECTION WITH DIPOLE-DIPOLE ARRAY  
 $P1P2 = C1C2 = 1 \text{ Dx}$   $n = 1-39$



MODEL: Target: Horizontal prism at depth 5Dx  
 Dimensions of the prism  $2 \times 2 \times 20 \text{ Dx}$   
 Resistivity of the prism 2000 Ohmm,  
 Resistivity of the environment: 500 Ohmm  
 Chargeability of the prism: 200 mV/V  
 Chargeability of the environment: 1 mV/V

Prof. Dr. A. Frasheri  
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Fig. 10

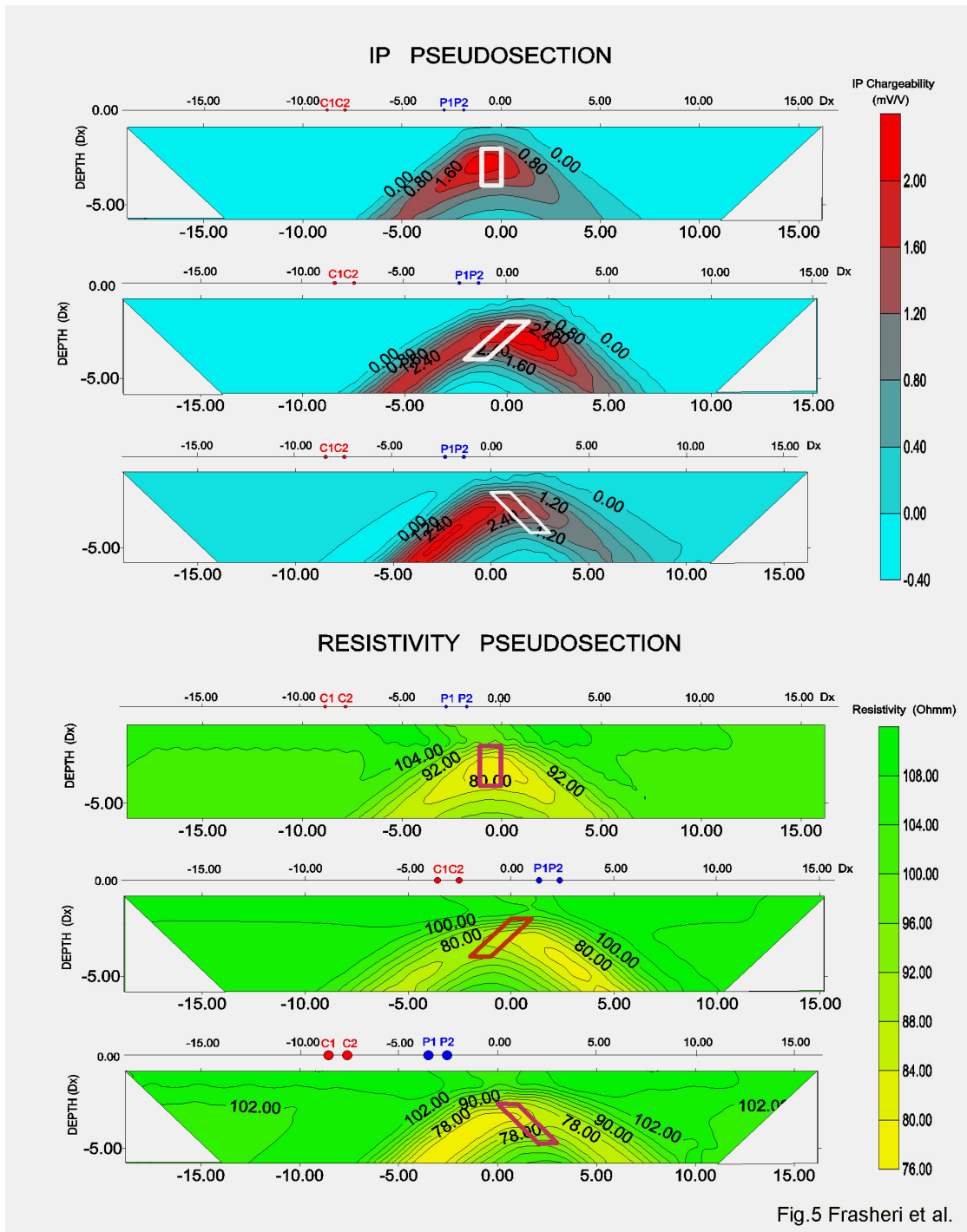


Fig. 11. IP and Resistivity Pseudo Section with dipole-dipole array,  $C_1C_2-P_1P_2=1$  Dx,  $n=1-11$  Dx. Mathematical model: 2D vertical prism at depth 1 Dx, dimensions of the prism section 4 x 50 Dx. Resistivity of the prism 3 Ohmm, IP Chargeability 50 mV/V, Resistivity of the environment 1,000 Ohmm, IP Chargeability of the environment 0.01 mV/V.

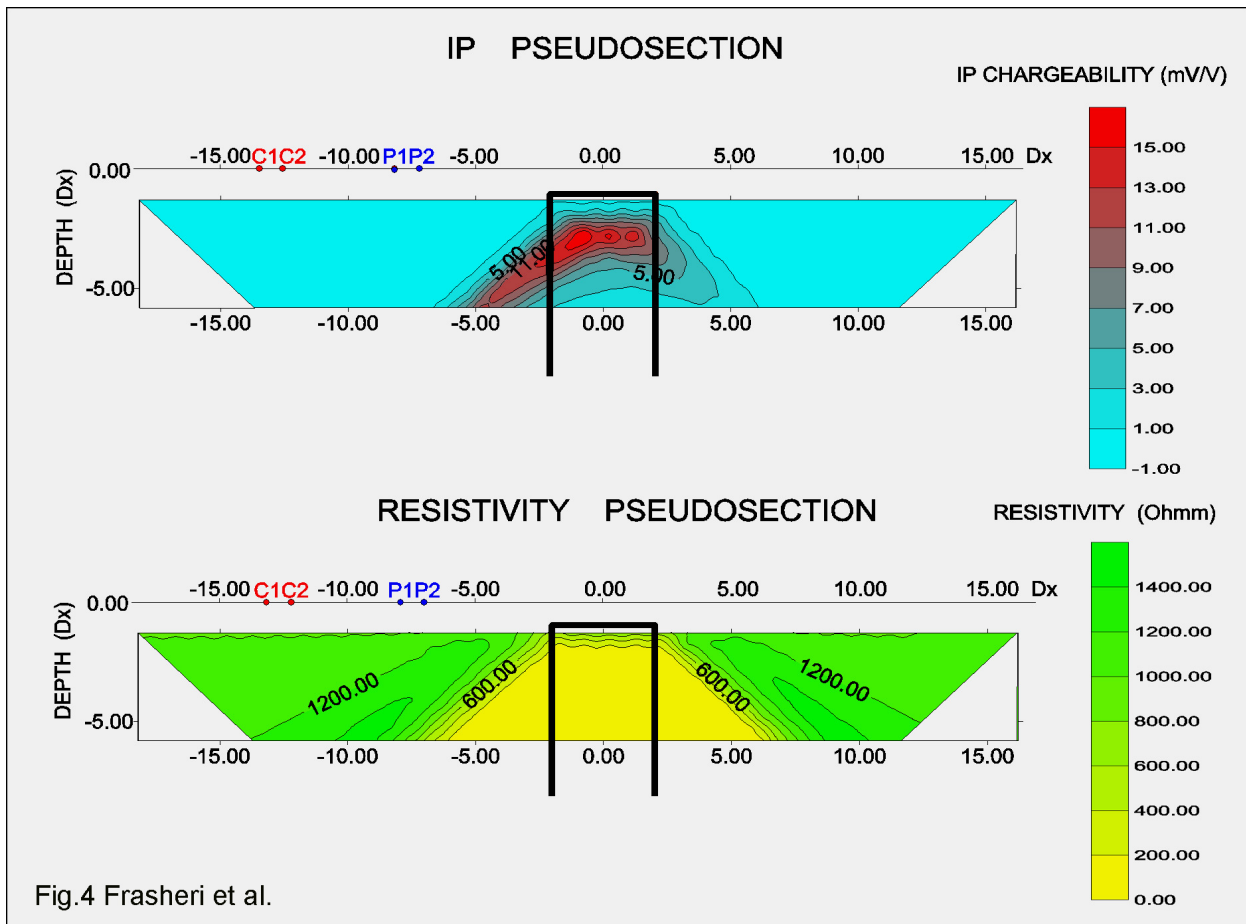


Fig. 12. IP and Resistivity Pseudo Section with dipole-dipole array.  $C_1C_2-P_1P_2=1$  Dx,  $n=1-11$  Dx. 2D Mathematical model. Dimensions of the prism section  $1 \times 2$  Dx. Resistivity of the prism 1 Ohmm, IP Chargeability 300 mV/V, Resistivity of the environment 100 Ohmm, IP Chargeability of the environment 0.01 mV/V.  
 a) vertical prism at depth 2 Dx, b) Inclined prism at depth 2 Dx, Western dip.  
 c) Inclined prism at depth 2 Dx, Eastern dip.

The response becomes more complicated when several targets are located under the surveying line. For a situation with two parallel polarizable inclined prisms like that in figs. 11 and 12, both  $C_1C_2P_1P_2$  and  $P_2P_1C_2C_1$  dipole-dipole arrays obtain a single IP anomaly in the centre and present some differences in contours shape (Fig. 13-a, b). A formal interpretation or even an inversion on these results cannot outline the presence of two distinct targets.

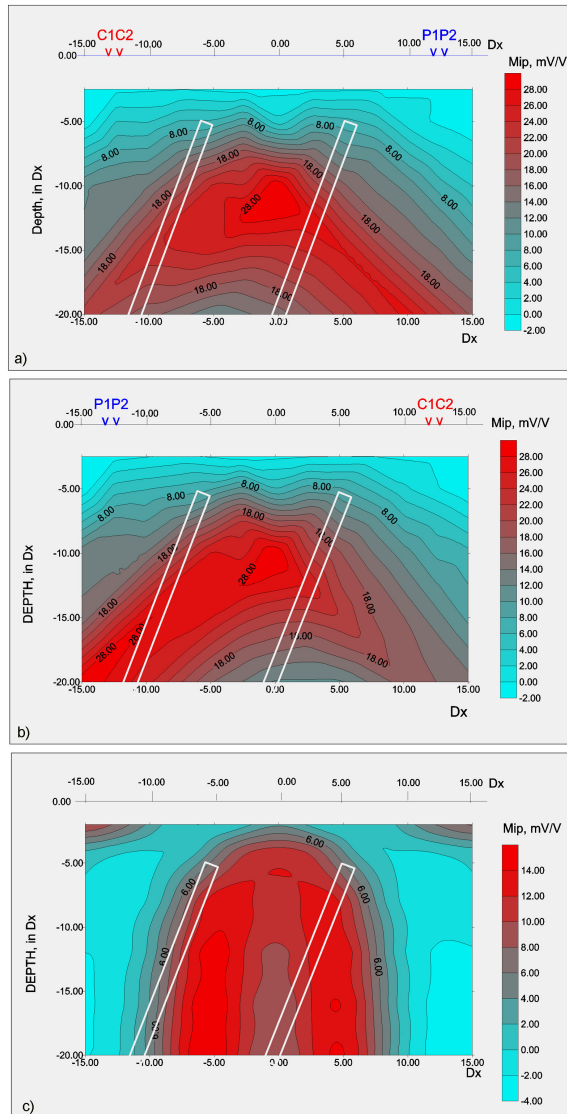


Fig.8 Frasher et al.

Fig. 13. IP Pseudo Section with dipole-dipole array,  $C_1C_2=P_1P_2=1$  Dx,  $n=1-39$ .

Mathematical Model: Two parallel inclined prisms ( $\text{dip}=70^\circ$ ) at depth 5 Dx, dimensions of the prisms 1 x 20 x 20 Dx. Distance between the prisms 10 Dx, Resistivity of prisms 2000 Ohmm, IP Chargeability 500 mV/V, Environment: Resistivity 500 Ohmm , IP Chargeability 0.01 mV/V.

- a) Dipole-dipole  $C_1C_2=P_1P_2$ ,
- b) Dipole-dipole  $P_1P_2=C_1C_2$
- c) Real Section with multiple gradient arrays.

#### 4. REAL SECTION

Limitations that are traditionally in traditionally configurations, as ex. “pseudosection” of the dipole dipole survey, has been overcome by gradient Realsection (Alikaj P. et al. 1981, ). These limitations, that have been presented in the paragraph 3, existed with respect to location, resolution and depth of investigation, inherent in conventional configurations. The IP/Resistivity Realsection is a technique that employs the data acquisition from multiple gradient arrays or Schlumberger vertical soundings to provide a presentation that is close to real distribution of the geo-electrical parameters in a geologic section. It is not a mathematical inversion but rather a presentation of the physical measurements in compliance with general distribution of the electrical field at depth. Algorithm developed in conjunction with these configurations, based on scale and mathematical modeling as well as orientation surveys over known deposits, allow presentation and interpretation of realsection technique in relation to real depth and location.

‘Realsection IP’ has come significant advantages over standard, double dipole or pole-dipole IP surveys, especially in depth of investigation and resolution. By using a short potential electrode distance (MN) the Realsection technique simultaneously provides with a high resolution of the near-surface IP/Resistivity targets (10-25 m) and a depth exploration capability (several hundred meters), this is not possible with conventional arrays.

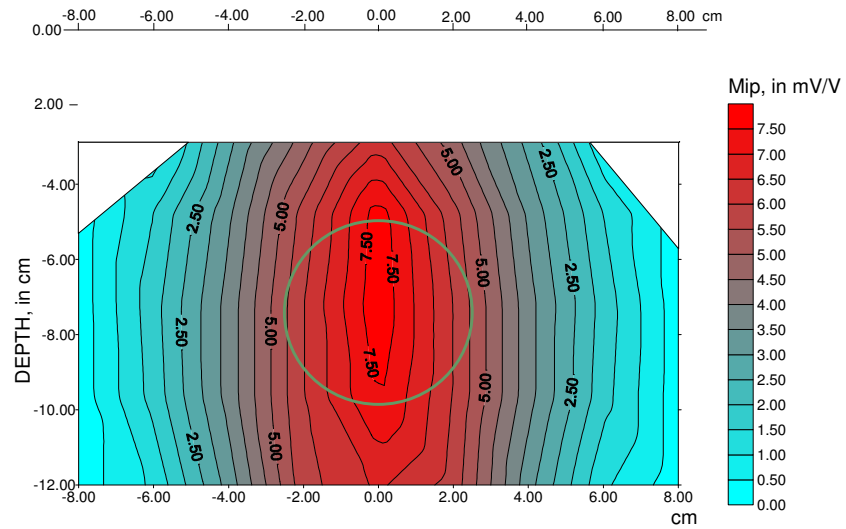
This method is turned to a specific depth of investigation by careful analysis of the geology and target model. The most common configuration is a gradient array. The first pass survey identifies zones worthy of detailed follow-up. Each subsequent pass build the section from depth to surface. The survey is continually refined in the field to concentrate the detailing on the anomalies with the highest potential of exploration. ‘Realsection IP’ has overcome the difficulties with gradient arrays by providing vertical resolution. A significant advantage over the common array geometrics of pole-dipole and dipole-dipole is that an increase in depth of resolution can be incremented logarithmically or semi-logarithmically as opposed to arithmetically. ‘Realsection IP’ can be applied to sub-vertical or sub-parallel structures with equal effectiveness.

‘Realsection IP’ is gaining acceptance for its ability to define depth narrow structure and for presenting this data in a section plotted at a depth that is calculated from the data and accurate to within 15% (Alikaj P., Gordon R., ).

In the fig. 14-22 are presented ‘Realsection IP’ of 2D Physical Modelling with gradient array for different shape polarizable targets, and 3D mathematical modeling of IP Realsections.

Mathematical model with IP Realsection array (Alikaj 1981, Langore Alikaj and Gjovreku 1989, Lubonja, Frasherri and Alikaj 1994) over the same targets, however, provides a different picture with two distinct anomalies (Fig. 13-c).

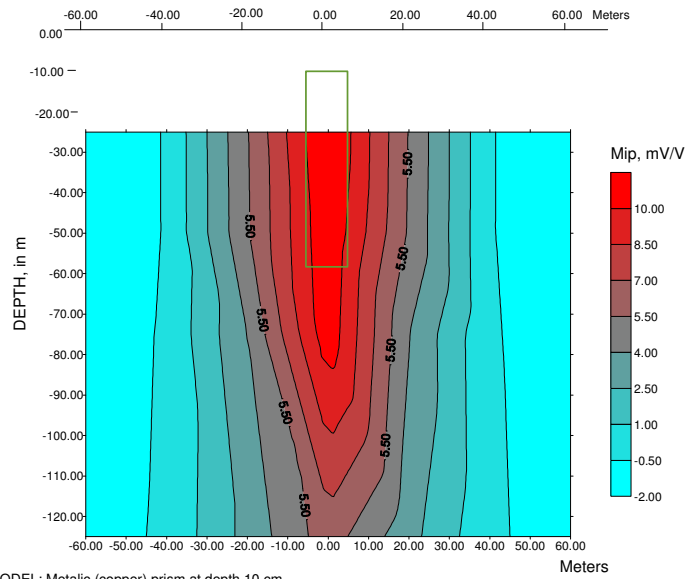
IP 2D PHYSICAL MODELING  
REALSECTION WITH GRADIENT ARRAYS



MODEL: Metallic (copper) cylinder at depth 5 cm  
Cylinder radius: 2.5 cm  
Surrounding environment: water

Prof. Dr. P. Alikaj  
July 1981

IP 2D PHYSICAL MODELING  
REALSECTION WITH GRADIENT ARRAYS



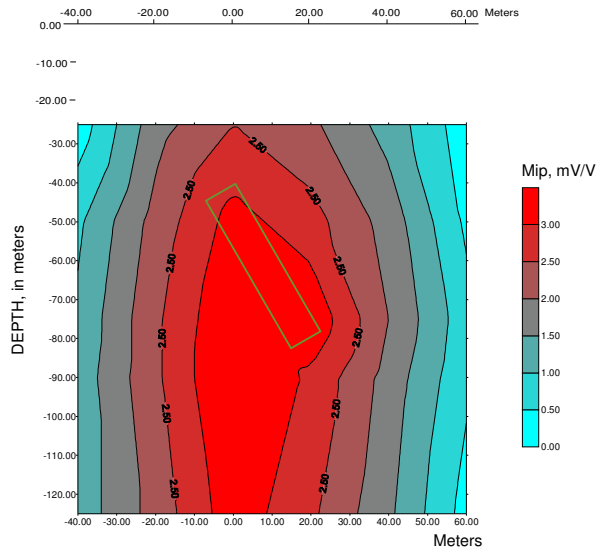
MODEL: Metallic (copper) prism at depth 10 cm  
Dimensions of the target: 10 x 50 m  
Surrounding environment: water

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July 1981

Fig. 14



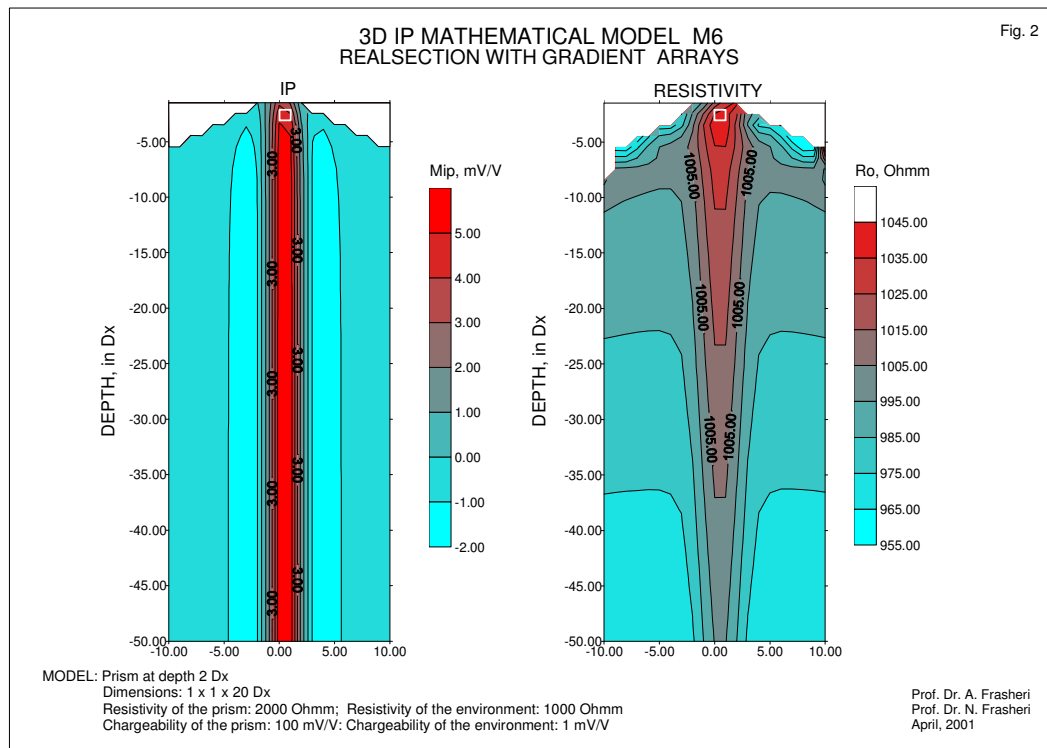
IP 2D PHYSICAL MODELING  
REALSECTION WITH GRADIENT ARRAYS



MODEL: Metallic (copper)inclined prism (dip=45o) at depth 10 cm  
Dimensions of the target: 10 x 50 m  
Surrounding environment: water

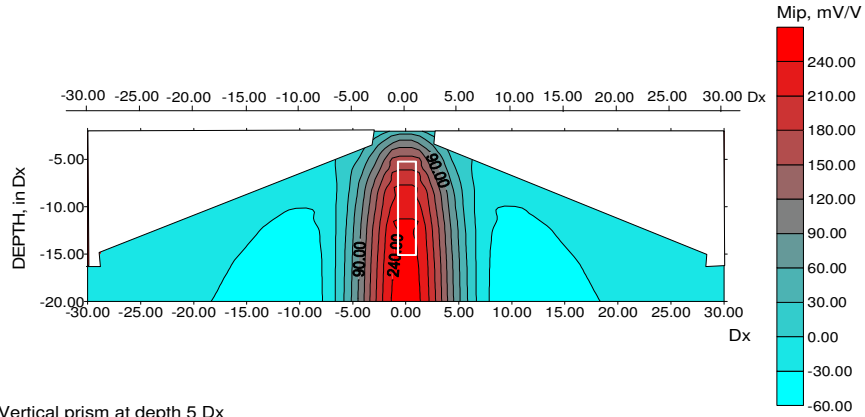
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July 1981

**Fig. 15**



**Fig. 16**

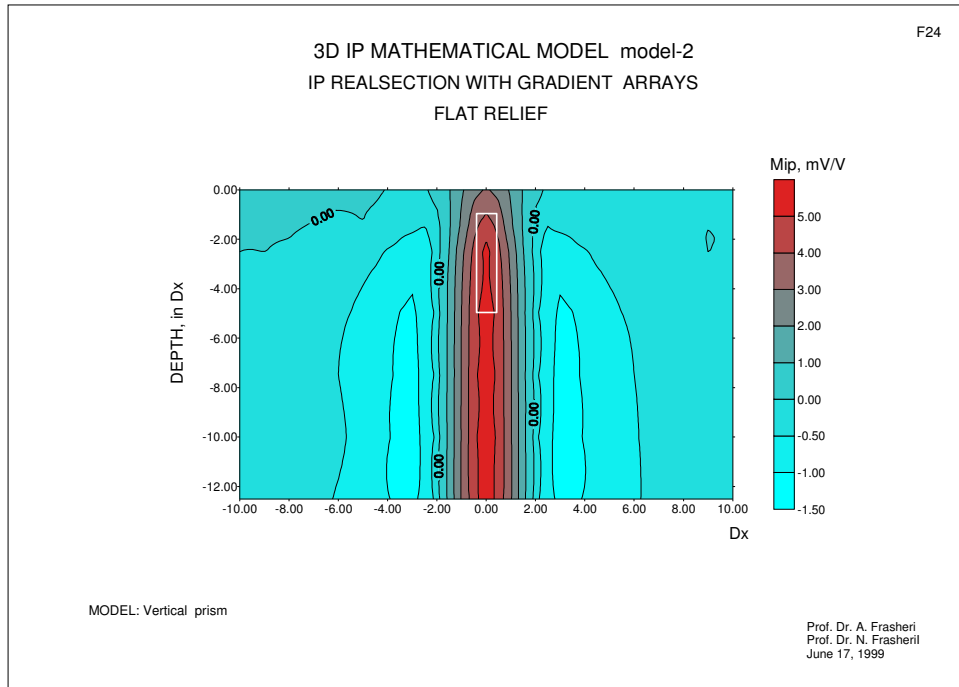
3D IP MATHEMATICAL MODEL Real10  
 IP REALSECTION WITH GRADIENT ARRAYS



MODEL: Vertical prism at depth 5 Dx  
 Dimensions: 2 x 10 x 20 Dx  
 Resistivity of the prism: 10 Ohmm  
 Resistivity of the environment: 500 Ohmm  
 Chargeability of the prism: 500 mV/V  
 Chargeability of the environment: 1 mV/V

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 June 1, 2001

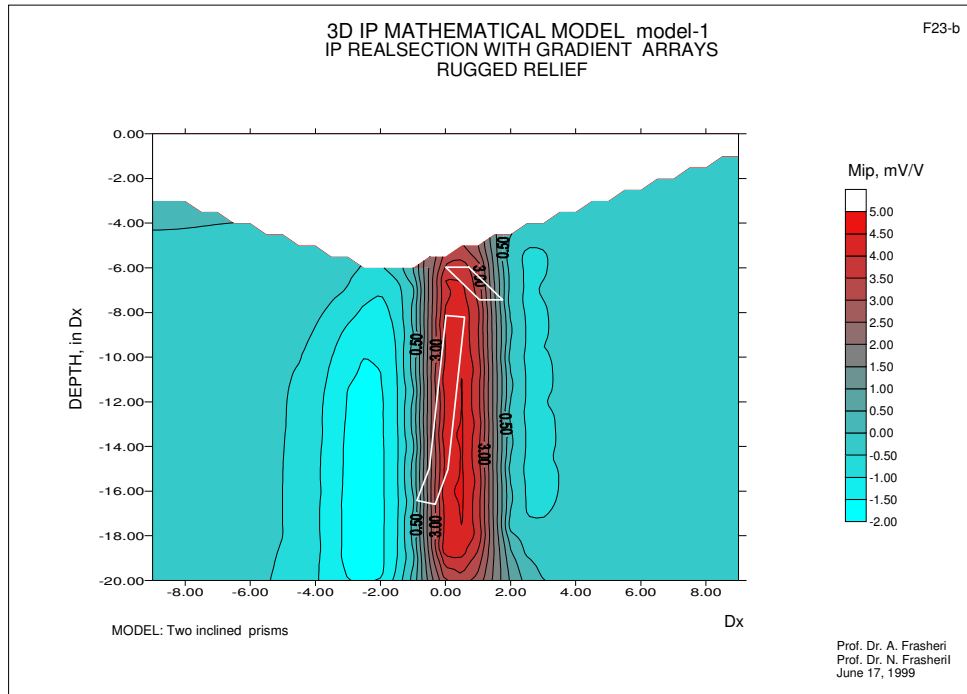
**Fig. 17**



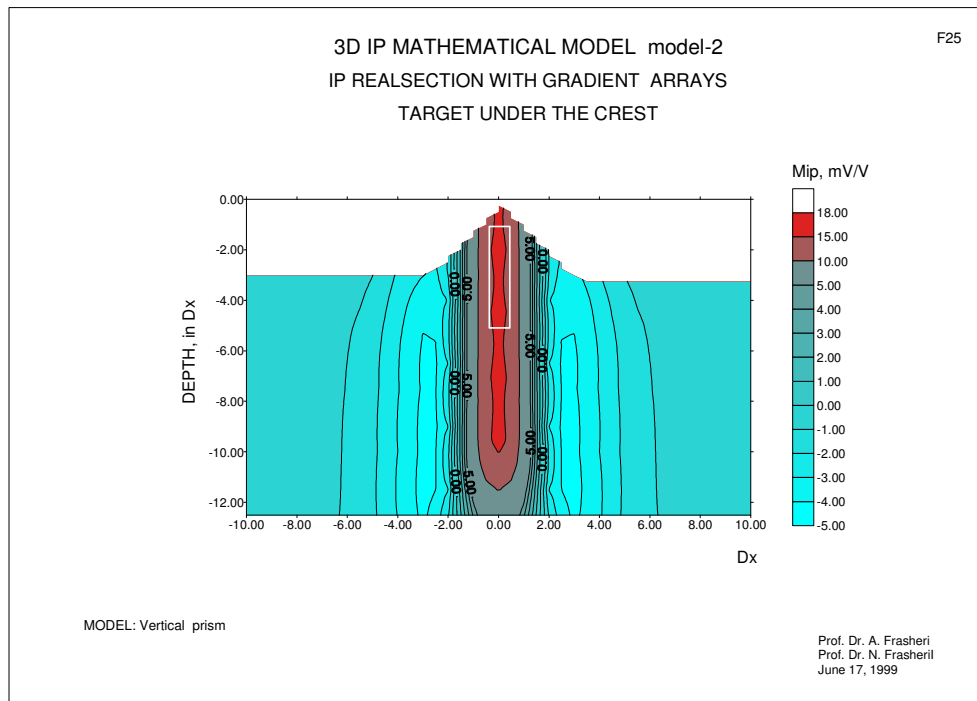
MODEL: Vertical prism

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 Prof. Dr. N. Frasher  
 June 17, 1999

**Fig. 18**

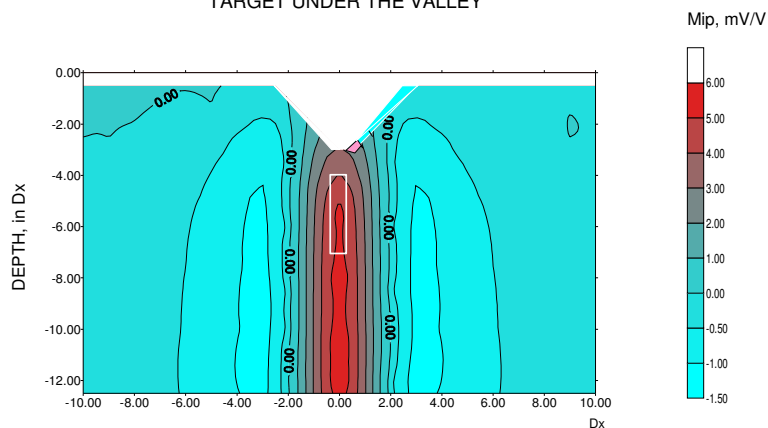


**Fig. 19**



**Fig. 20**

IP REALSECTION WITH GRADIENT ARRAYS  
 3D IP MATHEMATICAL MODEL model-2  
 TARGET UNDER THE VALLEY

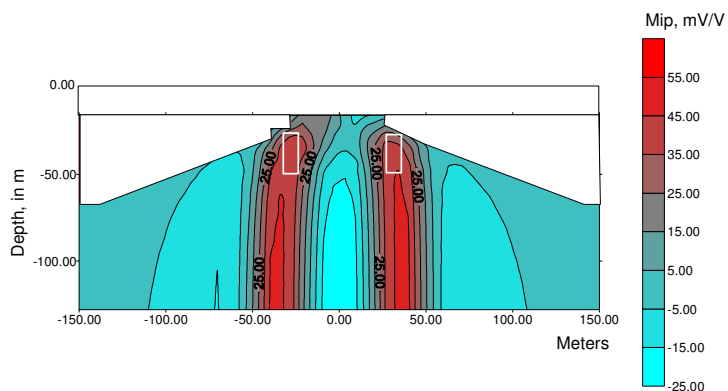


MODEL: Vertical prism

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 June 17, 1999

**Fig. 21**

3D IP MATHEMATICAL MODEL model-8  
 IP REALSECTION WITH GRADIENT ARRAYS



MODEL: Two vertical prisms at depth 25 m  
 Dimensions of the prisms: 10 x 25 x 200 Dx  
 Distance between the prisms: 50 m  
 Resistivity of the prisms: 100 Ohmm  
 Resistivity of the environment: 1000 Ohmm  
 Chargeability of the prisms: 500 mV/V  
 Chargeability of the environment: 1 mV/V

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 May 24, 2001

**Fig. 22**

## 5. SOME CONSIDERATIONS ON SO CALLED “IP DATA INVERSION”

The calculation of IP effect is based on the formulae of Bleil [Bleil D., 1953; Seigel H.O., 1959]:

$$U_{IP} = c \cdot \int_V \nabla U \cdot \left( \frac{1}{\bar{R}} \right) \cdot dv \quad (1)$$

Where:  $U_{ip}$  is the IP potential;

$\bar{R}$  is the distance vector from the integration point to the receiving point;

$\nabla U$  is the potential gradient of the primary electrical field, calculated by solving the finite element model.

The evaluation of Komarov is used for both modeling and inversion of IP data, supposing a formal similarity of environment polarization with the increasing of its electrical specific resistivity [Komarov V.A., 1972]:

$$C(U_0 + U_{ip}) \approx C U_0 \quad (2)$$

Where:  $U_0$ - potential of the polarizing current field,

$U_{ip}$ - potential of the IP fiend

C- IP susceptibility

In all calculations, the effect of IP is supposed as linear. Such modeling and inversions of IP pseudo-sections, carried out by many authors, have been a step forward for the interpretation of IP survey data and for the evaluation of IP methods. But new facts on the non-linear nature of IP phenomenon, together with results of mathematical and physical modeling of last ten years, arise new problems with regard to P modeling and inversion. If these problems will remain unsolved, it would decrease the effectively of IP investigations.

Conception of IP as a linear phenomenon and its usage in equations of modeling and inversion creates several characteristics in the configuration of calculated mathematically IP anomalies:

1. The upper parts of anomalies correspond with the upper sides of the polarized targets.
2. Anomalies remain open towards the depth, even below the bottom sides of targets. Continuation of IP anomalies below bottom sides of targets makes the interpretation difficult and its extension in depth as unsure. The presentation of anomalies is more complex, compared with pseudo-sections, for dipole-dipole and pole-dipole arrays. Migration of anomalies in pseudo-sections depends on the angle of inclination of targets and on the position of current and measuring electrodes relative to targets (there are left-arrays C1C2-P1P2 and right-arrays P1P2-C1C2). The reason of such configuration of IP

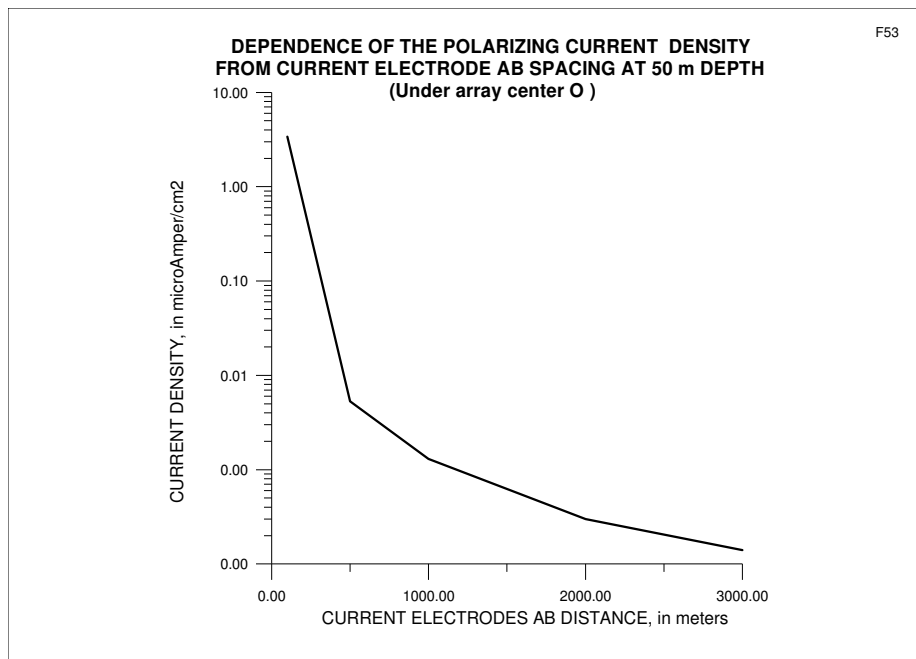
anomalies is due to the supposition, during mathematical calculations, that the IP has a linear dependence from the tension of polarizing electric field

Due to the different polarizing situations, IP phenomenon is characterized by:

1. Significant decrease of the intensity of polarizing electric field in depth. Increasing of investigation depth, different parts of the same target, as well as different targets in different depths, are situated in different polarizing conditions.
2. For the same depth of polarized targets, the intensity of polarizing electric field decreases by a great gradient relative when the distance AB of current electrodes increases (Fig. 23).

***Polarizing field voltage (E) at the depth 50 meters, in the medium with resistivity of 1000 Ohmm.***

Current Electrodes spacing [in meters]	Voltage of polarizing electric field [in mV/m]
100	33960
500	53
1000	13
2000	3
3000	1.4



**Fig. 23**

3. Depending on the environment, the voltage of polarizing field varies with a greater gradient. As result, the decrease of density and tension of polarizing fiend in depth means, because of non-linearity, less polarization compared with what is received by linear models. As result, in real sections from physical modeling the IP anomalies close under targets .
4. The effect of distributed IP is defined from survey arrays. This distribution is symmetric for gradient arrays, but asymmetric for dipole-dipole arrays making obligatory the inversion of IP data.

The stability and uniqueness of IP inversion solutions depend also from the application of a linear model for the IP phenomenon, but that is not quite true for the whole variation of applied polarizing tensions. As result, the lower part of polarized targets is unstable in IP inversions. It becomes more unstable when several targets are situated near each other, or in cases of targets near contacts between environments with different polarizability. The increase of depth of targets causes the increase of instability for inversion solutions and of its resolution capability (Fig. 24). Target shape, it's dimensions and depth of location are conditioned inversion results and stability, too (Fig. 25, 26, 27).

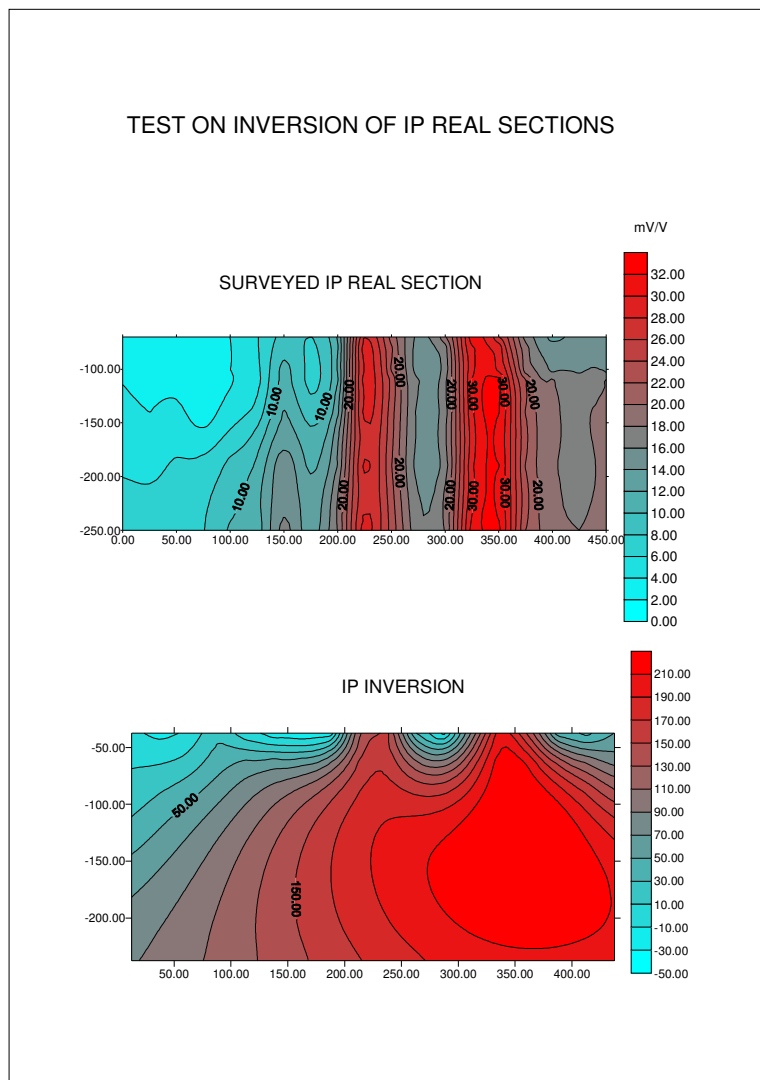
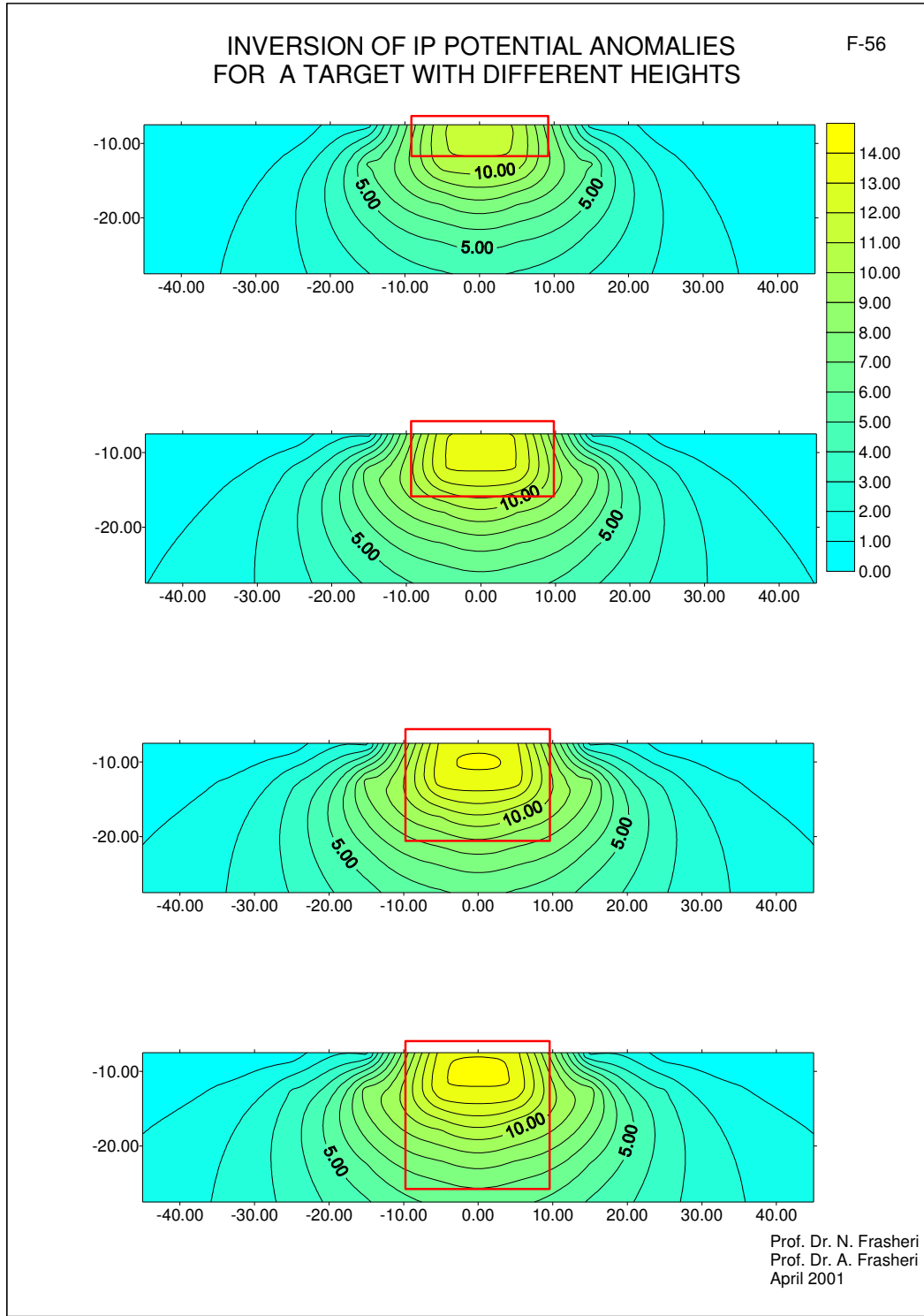
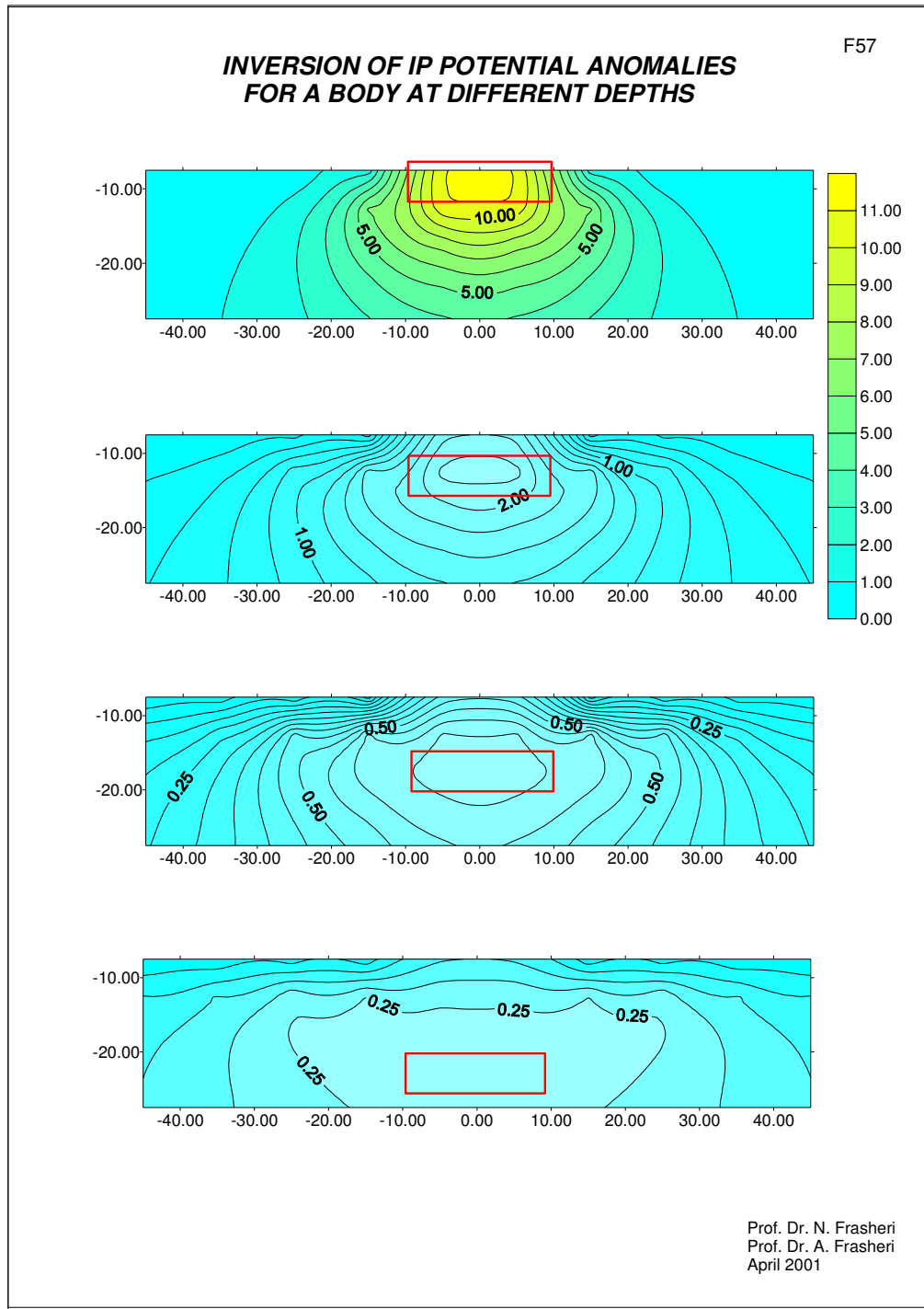


Fig. 24





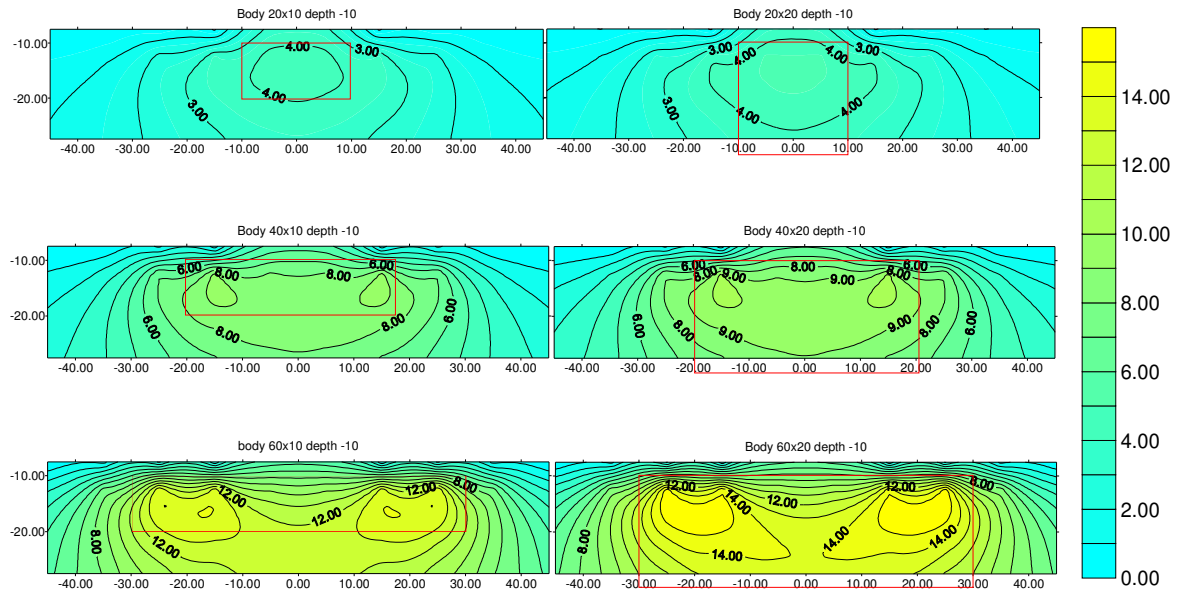
**Fig. 25**



**Fig. 26**

## IP TEST INVERSION

Norme SUM(||)  
Lagrange multiplier = 3



MODEL: Horizontal prism with different dimensions  
at depth 10 m and 20 m

Prof. Dr. N. Frasherri  
Prof. Dr. A. Frasherri  
April 2001

**Fig. 27**

## 6. CONCLUSIONS

1. The anomaly configuration in an IP/Resistivity survey with a dipole-dipole array is dependent on the location of the current and potential electrodes in connection to target. In this regard, logistical information about the survey should include the array orientation (left-array or right-array). The position of the array must be shown in plots and pseudosections. During the survey, it is necessary to keep the same orientation of current and receiving dipoles.
2. An accurate interpretation of IP/Resistivity data with dipole-dipole array should consider the information on electrode orientation on the survey line. The same recommendation is valid for the process of inversion interpretation.
3. Physical modeling of IP gives the proof that there are differences between real cases and mathematical models. In sections compiled with data from physical models the anomalies close under the bottom side of targets. In sections of mathematical linear

models IP anomalies remain open in depth, contrary to those of apparent resistivity. It is due to the fact that in used mathematical formulas the IP chargeability is considered as a linear phenomenon in the whole range of variation of polarizing tension.

4. The use for the inversion of formulas based on the linear IP phenomenon implies errors in compilation of sections based on approximation of inversed data. These errors may be comparable with the instability of the inversion itself.
5. To achieve the levels of actual requirements for the quality of IP surveys, it is necessary to well evaluate the non-linear character of IP phenomenon. It would permit a better conception of mathematical basis of IP, as well as a better match with the real situation of the phenomenon in nature. Used with the IP inversion, these new mathematical non-linear equations would permit more exact results as compared with the instability and non-uniqueness of inversion solutions.
6. An effective tool for exploration has been and continues to be 'Realsection IP'

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