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Finite element modelling of IP anomalous effect from bodies of any shape located in rugged relief area

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Abstract: *Finite element modelling of geoelectrical sections is carried out in two cases: (a) Ore bodies with a massive texture and an electrical resistivity in contrast with the resistivity of the surrounding rocks, where 2.5D modelling is applied; (b) Ore bodies with a disseminated texture having no contrast of resistivity with the surrounding rocks, where 3D models are under use. Two parameters characterize the constructed model - the apparent resisitivity and the induced polarization. The effect of the relief as well as of the global geological structure is taken into account. Case studies shown demonstrate different effects and usability of the modelling by finite elements.*

The results of such a modelling are presented according to a new method of "geoelectrical real section", proposed by Perparim Alikaj, and developed recently in QUANTEQ IP Inc., Canada. They are a synthesis of many year's works of the authors in collaboration with this company in a number of projects.

Key Words*: IP modelling, Resistivity modelling, Finite element modelling, Realsection*

INTRDUCTION

Resolving geophysical problems means a finite iteration of the couple interpretation ↔ modelling. Theoretical models exist for a number of ideal cases, rarely found in nature. The problem becomes more complicated when the depth of investigation increases, together with an increase in the secondary effects caused by the relief and the geological inhomogeneity in depth. In this paper the problem of modelling geoelectrical real sections is treated using finite elements to solve elliptic equations in a heterogeneous medium related to complex geological situations and rugged relief. This procedure is used both for resistivity and IP modelling.

PRINCIPLES OF APPLICATION OF FINITE ELEMENTS IN MODELLING GEOELECTRICAL SECTIONS

The key for modelling of geoelectrical sections is the scattering of an electrical field in a heterogeneous geological medium under a rugged relief. For this purpose we have used (Frasheri *et al*., 1984; Frasheri *et al*., 1990-94) the elliptic equation in its generalized form, which is related to the following weak problem (Zienkiewicz, 1977):

$$\min \int_V [(\nabla W)^T \mathbf{D} \nabla U - WQ]\, dv = \int_{S_n} w\, [\mathbf{n}^T \mathbf{D} \nabla U - U_n]\, ds \qquad (1)$$

where U is the electrical field potential; W, w are weight functions; $\boldsymbol{D}$ is the resistivity matrix; $\boldsymbol{n}$ is the unitary normal vector to the boundary S_n; U_n is the Newman boundary condition value; Q is the distributed electrical charge within the volume V.

We solved this problem by using parametric finite elements with four nodes. Normally, the geoelectrical section may be considered as a rectangle, the upper part of which is deformed in correspondence with the relief (Fig. 1).

The boundary conditions are of the Newman type, which present power electrodes positioned in two nodes of the upper edge of the rectangle. In the other part of the boundary we normally use zero Newman conditions.

The solution to the problem (1) gives the scattering of the electrical potential in a discrete form. These data need to be interpreted in the right way so as to give information compatible with that collected during field surveys. We had considered two parameters, the Apparent Resistivity (AR) and the Induced Polarization (IP).

MODELLING OF APPARENT RESISTIVITY ANOMALOUS EFFECTS

The meaning of "apparent resistivity" as a ratio between two "resistivities" is related to the formula:

$$\rho_a = R_o.\Delta U/\Delta U_o \quad (2)$$

where: ΔU is the difference between two adjacent stations of the electrical potential of heterogeneous geoelectrical section; ΔU_o is the corresponding difference of electrical potential of the homogeneous half-space, R_o is the resistivity in Ohm.m of the homogeneous half-space.

During the field measurements U_o is evaluated by the theoretical formula of an electrical dipole:

$$U_o = c\,(1/R_a - 1/R_b) \quad (3)$$

where U_o is the electrical field potential in the case of homogeneous half-space; R_a, R_b are distances from the calculation point to the current electrodes A and B; $c = R_o.I/2\pi$, where R_o is the resistivity and I is the current intensity.

We used two solutions to carry out mathematical modelling. First we solved the weak problem two times, once for the heterogeneous case and the other for the homogeneous half-space, thus having both discrete approximations of U and U_o in formula (2). The second solution was based on a special treatment of the boundary conditions. The real geoelectrical section has to be considered similar to the lower half infinite space, but the ordinary finite element model implies a finite domain as shown in Figure 1. As a consequence the application of the theoretical solution for U_o gives deformed results having a non-negligible error. To avoid this error it is necessary to imply "the infinite" on the finite boundary.

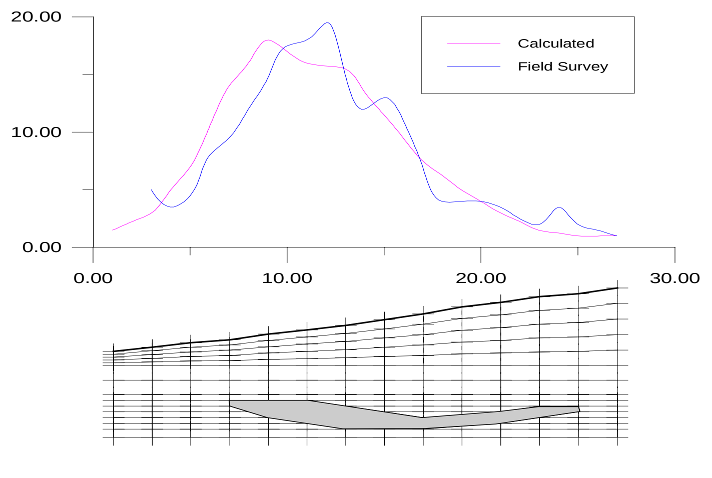

FIG. 1. A finite element section of a geoelectrical section in a rugged relief, IP is given in percentage.

A "classical" solution of this problem is the use of "infinite elements". A case of infinite elements used for geoelectrical models is treated by Frasheri (1983). Another solution, based on hybrid elements and Fourier transform, is given by Tong and Rossettos (1978). We used a simple solution for geosections having no important heterogeneous horizontal layered structure. In this case it is possible to evaluate theoretically the normal gradient of the potential at the boundary points using the formula:

$$Du/dn = c\,(\mathbf{R}_a/\mathbf{R}_a^3 - \mathbf{R}_b/\mathbf{R}_b^3).\mathbf{n} \quad (4)$$

where dU/dn is the normal to the boundary gradient of electrical field potential; R_a, R_b are distance vectors of the boundary point to the current electrodes A and B, while R_a, R_b are the respective distances; $\boldsymbol{n}$ is the normal unitary vector at the boundary points

In this case we can simply calculate the flux of electricity on the boundary nodes and add respective values in the right side of the simultaneous linear equations resulting from the problem (1):

$$\mathbf{K}.\mathbf{U} = \mathbf{F} \quad (5)$$

where $\boldsymbol{K}$ is the master matrix of the system; $\boldsymbol{U}$ is the vector of discrete values of the potential U in the nodes; $\boldsymbol{F}$ is the vector of flux concentrated in the boundary nodes.

A comparison of apparent resistivity values over a geosection with a vertical contact for the theoretical, finite elements and "infinite" elements is given in Figure2.

MODELLING OF INDUCED POLARIZATION ANOMALOUS EFFECTS

We used the finite element modelling of IP in two ways, related to the physical characteristics of the geosections. The IP phenomenon is modeled mathematically as the potential of a double layered surface, which represents the boundary between the mineralized homogeneous ore body and the surrounding rocks. In reality mineralized ore bodies have a certain texture, which is not actually homogeneous. Bodies with a disseminated texture have their IP scattered in the volume, and bodies with a massive texture have the IP concentrated on their boundary surface. The calculation of the IP effect is based on the formula of Bleil (Bleil, 1953; Seigel, 1959), as well as on Komarov's evaluation (Komarov, 1972) assuming that $C(U_o+U_{ip}) \approx CU_o$.

Taking into account the fact that in some cases the heterogeneity of the medium may influence considerably in the IP responses measured on the earth surface, we used 2.5D finite element modelling of IP for a heterogeneous medium (Fig. 1). After calculating the potential U of the electrical field, we used the Bleil formula for the calculation of the IP effect:

$$U_{ip}= c \int_V \nabla U \, (1/R) \, dv \qquad (6)$$

where U_{ip} is the potential of the IP field; R is the distance vector from the integration point to the observation point; ∇U is the potential gradient of the primary electrical field, calculated by solving the finite element model.

For 3D modelling of bodies with a massive texture in homogeneous medium we used the Bleil formula after its transformation using Green's formula:

$$U_{ip}= c \int_S (1/R) \, (dU/dn) \, ds \qquad (7)$$

where R is the distance vector from the integration point to the observation point; dU/dn is the gradient of the primary electrical potential on the boundary S of the body, calculated as in formula (4).

The integral is numerically calculated by using the concept of finite elements for the boundary of the body, and by using standard numerical integration methods for the finite elements, defining automatically the number of integration points on the basis of the relative size of elements (Fig. 3).

Being already a classical theory, finite elements continue to give way to new aspects of development and application of geophysics. Finite element modelling of complicated geological situations is necessary not only as a proof of the correctness of the field data interpretation. It is also an important factor for the development of a new concept and respective techniques, as the "real section" (Langore, 1989), and special methodologies for the field surveys. A typical IP real section modelling is presented in Figure 4.

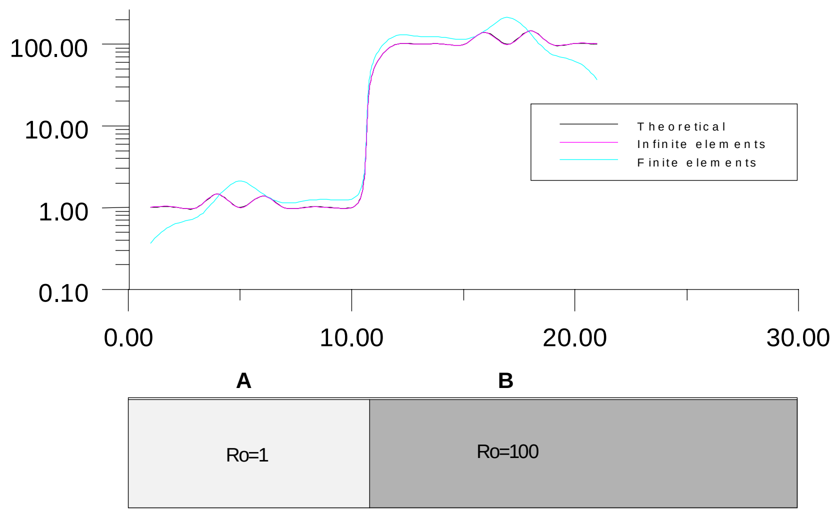

FIG. 2. Comparison of theoretical, finite elements and "infinite" elements solutions for the apparent resistivity anomaly in Ohm.m over a vertical contact with resistivity contrast 1:100 Ohm.m.

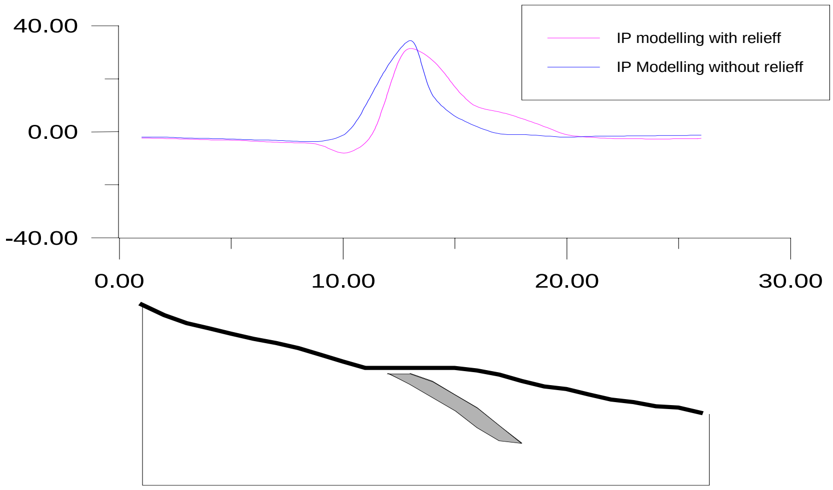

FIG. 3. A 3D modelling of the IP anomalous effect in percentage.

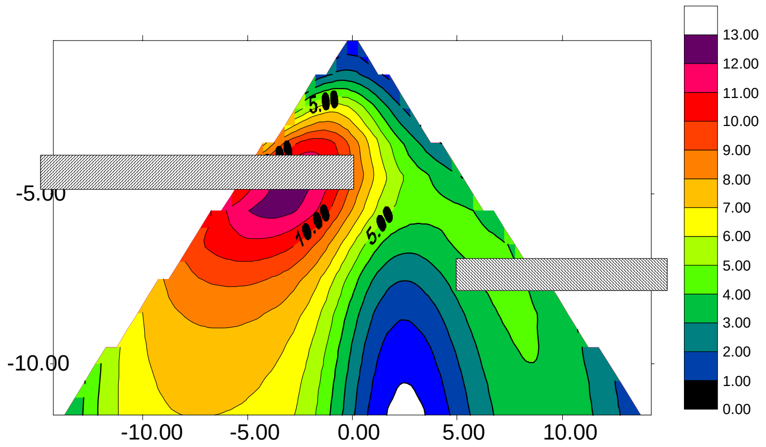

FIG. 4. IP real section of two layers in different depths, IP in percentage.

CONCLUSIONS

Finite elements are a good tool for the modelling of complicated geoelectrical sections, which are characteristic of the Albanian geology. In a number of cases it permitted to evaluate correctly the influence of the rugged relief effects and the geological conditions in layered mediums, contacts and faults on the anomalies of ore bodies or mineralized zones.

Real geoelectrical sections, which are created by using the methodology also presented in this paper, offer a reliable field data interpretation. Moreover, real sections have shown the existence of many problems related to the interpretation of field data, as well as the need for special studies for solving these problems.

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